is solvable if and only if

$$\rho_m(M) \equiv \sum_{i=0}^{k-m} M^{p^{ni}} \frac{L_{m+i-1}}{L_i} \equiv 0.$$

It will be remarked that in general the first criterion is more useful. However, the latter is of some interest in itself. Furthermore, it suggests possible criteria for more general classes of congruences; I hope to develop the matter elsewhere.

DUKE UNIVERSITY

A DIFFERENTIAL EQUATION FOR APPELL POLYNOMIALS†

BY I. M. SHEFFER

By a *set* of polynomials $\{P_n(x)\}$, $(n=0, 1, 2, \cdots)$, we shall mean an infinite sequence in which $P_n(x)$ is of degree *exactly*‡ n. Corresponding to a given set $\{P_n\}$ there are infinitely many sequences of polynomials $\{L_n(x)\}$ (with $L_n(x)$ of degree not exceeding n) and sequences of numbers $\{\lambda_n\}$ such that $\{P_n\}$ satisfies the linear differential equation (usually of infinite order) with parameter:§

$$(1) \qquad L[y(x)] \equiv \sum_{n=0}^{\infty} L_n(x) y^{(n)}(x) = \lambda y(x),$$

which for $\lambda=\lambda_n$ gives $P_n(x)$. In fact, suppose $\{P_n\}$ is given. Let $\{\lambda_n\}$ be any sequence of numbers subject only to the condition that λ_n is not identically zero in n. Then a unique sequence $\{L_n(x)\}$ exists such that $L[P_n(x)]=\lambda_n P_n(x)$, $(n=0, 1, \cdots)$, where not all the L_n's are identically zero, and where no $L_n(x)$ is of degree exceeding n. The polynomial $L_n(x)$ is readily obtained by recurrence from $L_0, \cdots, L_{n-1}$. If we write

$$(2) \qquad L_n(x) = l_{n0} + l_{n1}x + \cdots + l_{nn}x^n,$$

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† Presented to the Society, April 25, 1935.

‡ For many purposes it suffices to have $P_n(x)$ of degree not exceeding n. Here, however, it is convenient to use the stricter condition.

§ See Sheffer, American Journal of Mathematics, vol. 53 (1931), pp. 29–30, for a relation suggestive of (1).