ON THE COMPLEX ROOTS OF ALGEBRAIC EQUATIONS*

BY A. J. KEMPNER

1. *Introduction.* The topic I have chosen is so vast that a systematic treatment in an address is impossible. I shall, therefore, restrict my report to some phases of the problem to which I have made contributions, and to such papers as deal with related questions and which have come to my notice.

I sincerely appreciate this opportunity to bring together the results of several articles, all dealing more or less directly with the problem of complex roots, but scattered in various journals over a period of many years.

Our equations are always polynomials, equated to zero, and, unless otherwise stated, with real coefficients. The notation $f_n(z)=0$ indicates that the equation is of degree n. Merely to answer the question whether equations of high degree are ever actually solved, it may be pointed out that a table of natural sines and cosines, say for every $1''$, is nothing but a complete tabulation of the solutions of the equation $x^{1296000}-1=0$.

2. *On Equations with Roots* $e^{i\theta}$. We have in the literature many theorems on equations with roots of absolute value unity. One of the most interesting of these is Kronecker's theorem:

If the coefficients of an equation are ordinary integers, if the coefficient of the highest power is unity, and if all roots are of absolute value unity, then all the roots are roots of unity, and the equation is therefore solvable by radicals.

A companion theorem is as follows.

With the same restrictions on the coefficients, if all roots are real and of absolute value <2, *the roots are all of the form* $2\cos \pi k$, *where* k *is rational.*

In two short notes† I have derived necessary and sufficient conditions that an equation have some, or all, of its roots of the form $e^{i\theta}$.

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* An address presented to the Society, by invitation of the program committee, Lincoln, Nebraska, November 30, 1934.

† Kempner, Archiv der Mathematik und Physik, (3), vol. 25 (1916), pp. 236–242. Kempner, Tôhoku Mathematical Journal, vol. 10 (1916), pp. 115–117.