A SET OF DEFINING RELATIONS FOR THE SIMPLE GROUP OF ORDER 1092*

BY ABRAHAM SINKOV

1. *Introduction.* This paper is essentially a continuation of a previous paper, by H. R. Brahana, entitled *Certain perfect groups generated by two operators of orders two and three*,† the major portion of which is devoted to the relations $S^3 = T^2 = (ST)^7 = 1$. The groups defined by these relations are all perfect, that is, they coincide with their commutator subgroups. Consequently, the order of the commutator of S and T is included as an additional defining relation. The purpose of this note is to study the case when this commutator is of order 7 and to show that the relations

$$S^3 = T^2 = (ST)^7 = (S^{-1}T^{-1}ST)^7 = 1$$

completely define the simple group of order 1092. This result is then made use of to show that the non-alternating simple group of order 20,160 can not be generated by two operators of orders 2 and 3.

2. *Equivalent Generators.* It is shown in Brahana's paper that the generators S and T may be replaced by a pair of equivalent generators Q and R, in the sense that $\{S, T\}$ is the same as $\{Q, R\}$. The two sets of generators are connected by the relations

$$S = R^2Q, \qquad T = Q^{-1}R^4.$$

In terms of Q and R, the defining relations

$$S^3 = T^2 = (ST)^7 = (S^{-1}T^{-1}ST)^7 = 1$$

become

$$A: \qquad R^7 = Q^7 = 1, \qquad Q^{-1}R^4 = R^3Q, \qquad R^2QR^{-1} = Q^{-1}RQ.$$

The method of determining the order of the group G consists of an enumeration of the co-sets of G as regards the cyclic group

---

* Presented to the Society, February 23, 1935.

† American Journal of Mathematics, vol. 50 (1928), p. 345.