CYCLIC FIELDS OF DEGREE p^n OVER F OF CHARACTERISTIC p*

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1. *Introduction.* The theory of cyclic fields is a most interesting chapter in the study of the algebraic extensions of an abstract field F. When F is a modular field of characteristic p, a prime, particular attention is focussed on the case of cyclic fields Z of degree p^n over F. Such fields of degree p, p^2 were determined by E. Artin and O. Schreier.†

In the present paper I shall give *a determination of all cyclic fields Z of degree p^n over F of characteristic p.*

2. *Normed Equations.* An equation

$$\lambda^p = \lambda + a, \qquad (a \text{ in } F), \tag{1}$$

is called a *normed equation.* If x is any root of (1), then so are $x+1, x+2, \cdots, x+p-1$. Using this fact, Artin-Schreier have proved the following lemmas.

LEMMA 1. *A normed equation is either cyclic or has all of its roots in F. Every cyclic field of degree p over F may be generated by a root of a normed equation.*

LEMMA 2. *Let $F(x)$ be cyclic of degree p over F,*

$$x^p = x + a, \qquad (a \text{ in } F). \tag{2}$$

Then a quantity y of $F(x)$ which is not in F satisfies a normed equation if and only if

$$y = kx + b, \quad (k = 1, 2, \cdots, p;\ b \text{ in } F). \tag{3}$$

LEMMA 3. *Let c in Z have degree $t \leqq p-2$ in x. Then there exists a quantity $g = g(x)$ in Z such that*

$$g(x+1) - g(x) = c. \tag{4}$$

Moreover, g is uniquely determined up to an additive constant in F.

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† Hamburg Abhandlungen, vol. 5 (1926–7), pp. 225–231.