METRICALLY TRANSITIVE POINT TRANSFORMATIONS*

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1. *Introduction*. Let

$$(1)\qquad T^{(j)}:\qquad \xi_i^{(j)} = \xi_i^{(j)}(x_1, x_2, \cdots, x_n), \qquad (i = 1, 2, \cdots, n; j = 1, 2, \cdots),$$

denote a denumerable set of point transformations in n-dimensional euclidean space, the transformations of which leave a certain p-dimensional ($p \leqq n$) region R (region $\equiv$ open, connected point set) of this space invariant. The case where R is the entire n-dimensional space is not excluded.

The set of transformations (1) *will be said to be metrically transitive with respect to R, if the complement set* (*with respect to R*) *of every non-zero subset of R that is invariant under each transformation of the set* (1) *is a zero set.*† *If the transformations of the set* (1) *form a group, this group is said to be metrically transitive with respect to R.*

If the set (1) consists of a single transformation T and if $p=2$, the above definition is sensibly the same as that given by Birkhoff and Smith‡ for metrically transitive surface transformations. If the transformations of the set (1) are composed of the iterations of a single transformation T and its inverse T^{-1}, together with the identity transformation, the transformations of the set form an infinite cyclic group which is metrically transitive with respect to a certain region if, and only if, one of the elements of this group, exclusive of the identity transformation, is metrically transitive with respect to this region.

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† *Non-zero set* $\equiv$ a set of positive p-dimensional Lebesgue measure. *Zero set* $\equiv$ a set of zero p-dimensional Lebesgue measure. In this paper the word measure refers to Lebesgue measure; the measure of a measurable set S is denoted by mS.

‡ G. D. Birkhoff and P. A. Smith, *Structure analysis of surface transformations*, Journal de Mathématiques, vol. 7 (1928), p. 365.