FERMAT'S LAST THEOREM AND THE SECOND FACTOR IN THE CYCLOTOMIC CLASS NUMBER

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1. *Introduction.* As usual in the relation

$$x^l + y^l + z^l = 0, \tag{1}$$

where l is an odd prime and x, y, z are rational integers prime to each other and none zero, we shall refer to the case where xyz is prime to l as case I; if $xyz \equiv 0 \pmod{l}$ then we call this case II. I now give a sketch of a proof of a theorem which appears to be the principal result I have so far found concerning the first case of the last theorem.

THEOREM 1. *If* (1) *is possible in case* I, *then the second factor of the class number of the cyclotomic field defined by*

$$\zeta = e^{2i\pi/l}$$

is divisible by l.

From (1) in case I we have either

$$x + \zeta y = \eta\omega_1\omega_2 \cdots \omega_s\alpha^l, \tag{2}$$

or

$$x + \zeta y = \sigma\beta^l, \tag{2a}$$

where η and σ are units, α is a number, and β is an integer in $k(\zeta)$, while each ω is the lth power of an ideal in $k(\zeta)$, not a principal ideal, and, as shown by Pollaczek,*

$$\omega_i^{s-r\alpha_i} = \delta\gamma^l,$$

where δ is the unit in $k(\zeta)$, a_i is in the set $1, 2, \cdots, l-2$, r is a primitive root of l and we are employing the Kronecker-Hilbert notation of symbolic powers, s denoting the substitution (ζ/ζ^r). We also have (Pollaczek†) for a_i even

$$\omega_i/\omega_{-i} = \delta_i\gamma_i^l,$$

* Mathematische Zeitschrift, vol. 21 (1924), pp. 19 and 22.

† Loc. cit., p. 22.