

KOJIMA ON DOUBLE DIRICHLET SERIES

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The object of this note is to direct attention to the most extensive and for us one of the most interesting papers by Tet-suzô Kojima, whose death on April 7, 1921, at the early age of thirty-four deprived Japan of an able and promising mathematician. This paper, which has only just now come to our notice* and whose existence may not be generally known, is *On the double Dirichlet series*, (Science Reports of the Tôhoku Imperial University, (1), (math., phys., chem.), vol. 9 (1920), pp. 351–400). It is devoted mainly to the convergence properties of double Dirichlet series, certain related double integrals, double factorial series, and double binomial coefficient series. Unaware of Kojima's work, Leja and the present writer† have recently considered some of the same questions; it may therefore be appropriate to give a brief description of Kojima's memoir and to point out the main features of its relationship to the more recent papers cited.

A double series $\sum_{i,j=1}^{\infty} a_{ij}$ for which the sequence $\{S_{mn}\}$ of partial sums, $S_{mn} = \sum_{i,j=1}^{m,n} a_{ij}$, converges as $m, n \rightarrow \infty$ and of which each row and column converges has been called *regularly convergent* by Hardy. If $\{S_{mn}\}$ is both convergent and bounded, the series may be said to be *convergent-bounded*. One sees at once that every series regularly convergent is convergent-bounded but that the converse is not true. The notion of regular convergence is basic in Kojima's work; that of convergence-bounded in L, AI, and AII.

* An obituary in the Tôhoku Mathematical Journal, vol. 19 (1921), 2 pp. following the table of contents, gives a complete list of Kojima's fifteen papers, including two published posthumously, together with an account (in Japanese) of his life.

† Leja, *Sur les séries de Dirichlet doubles*, Comptes-Rendus du 1 Congrès des Mathématiciens des Pays Slaves, Warsaw, 1930, pp. 140–158; Adams, *On multiple factorial series*, Annals of Mathematics, (2), vol. 32 (1931), pp. 67–82; and Adams, *Note on multiple Dirichlet and multiple factorial series*, *ibid.*, vol. 33 (1932), pp. 406–412. Hereafter these papers will be referred to respectively as L, AI, and AII.