

Since the discriminant of the cubic is a square, b can be chosen so that $\alpha' = 1$. The corresponding value of β' is

$$\beta' = \frac{-729\beta^4}{\alpha^3 b^3 (4\alpha^3 - 27\beta^2)}.$$

The ratio of β' to α' is $-9\beta^2/(\alpha^3 b)$. If we take α in (3) to be 1 and determine b in (20) so that α' is 1, we have

$$\beta' = -\beta(4 - 27\beta^2)^{1/2}.$$

We have therefore the following theorem.

THEOREM 3. *If p is of the form $6k-1$ and $x^3-x+\beta \equiv 0$ is irreducible, then $x^3-x+\beta(4-27\beta^2)^{1/2} \equiv 0$ is also irreducible.*

If β in the above theorem is not $1/3$, the second cubic is distinct from the first. By repeated applications of the theorem we obtain a set of cubics of the form $x^3-x+\beta \equiv 0$, but in general we do not obtain all of them.

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THE ALGEBRA OF SELF-ADJOINT BOUNDARY-VALUE PROBLEMS*

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1. *Introduction.* By algebraic processes, D. Jackson† obtained in matrix form the condition for self-adjointness of differential systems of any order. The purpose of this paper is to develop by means of the matrix criterion the explicit conditions for self-adjointness of the boundary conditions associated with self-adjoint and anti-self-adjoint differential equations.

2. *Even-Order Systems.* Let $L(u)$ denote the self-adjoint differential expression‡

$$(1) \quad L(u) \equiv (p_m u^{(m)})^{(m)} + (p_{m-1} u^{(m-1)})^{(m-1)} + \dots + p_0 u,$$

where m is any positive integer, $p_i(x)$ is of class C^i , and $p_m(x) \neq 0$ in the interval $(a \leq x \leq b)$. Along with

* Presented to the Society, October 31, 1931.

† D. Jackson, Transactions of this Society, vol. 17 (1916), pp. 418-424.

‡ Bounitzky, Journal de Mathématiques, (6), vol. 5 (1909), p. 107.