

AN INVERSE PROBLEM IN DIFFERENTIAL
EQUATIONS*

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1. *Introduction.* The differential equation as a tool requires no introduction to either the mathematician or the applied scientist. Problems in endless variety are continually solved through this medium, the process almost invariably beginning with an epitome of the problem's essential characteristics in the form of a differential equation, which is thus determined explicitly both as to its structure and its coefficients, and proceeding thence to a deduction of the form or properties of a suitable solving function.

The present note is devoted to a problem in which this customary order of events is in large measure reversed. The formulation of the problem yields in this case the structural form of a differential equation, and beyond this the existence of a solution which satisfies certain specified conditions. From these data the determination of the equation itself, that is, of its coefficient function, is required and constitutes the solution of the problem.

2. *The Physical Problem.*† In the investigation of shallow geological structures, and in the study of the electrical resistivity of the earth's crust at depths below the surface, an appropriate experimental procedure centers around the supply of a direct electric current through a small electrode to the surface of the earth. The electrical potentials which result at the surface of the earth are measurable at all distances from the electrode, and constitute entirely the immediately obtainable data. From them it is desired to compute, if possible, the conductivity of the earth below as a function of the depth.

With the idealizations involved in regarding the conductivity as a differentiable point function depending upon the depth alone, and in taking the ground as a horizontally uniform in-

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† The problem and its formulation as outlined in this section are due to Professor L. B. Slichter of the Massachusetts Institute of Technology. A complete and detailed discussion of it is given in a geophysical paper by L. B. Slichter, in *Physics*, vol. 4, Sept., 1933.