

SIMPLIFICATION OF THE SET OF FOUR POSTULATES FOR BOOLEAN ALGEBRAS IN TERMS OF REJECTION*

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1. *Introduction.* Some time ago, I presented† a set of four postulates for Boolean algebras expressed in terms of Sheffer's "stroke" operation, or the operation of "rejection." This set, which is a reduction of Sheffer's set of five postulates for Boolean algebras,‡ uses the stroke as the only primitive idea, besides that of *class*, and retains the characteristic of Sheffer's set of defining in terms of its primitives *all* the special Boolean elements, *zero*, the *whole*, and the *negative* of an element. It is fitting that this economical set of postulates should be as simple as possible. But this is not the case. In my effort to attain economy in the number of postulates, I paid too little attention to the matter of simplicity in statement of the postulates, with the result that one of the postulates, Postulate P_4 , is unnecessarily complex. It is my object now to offer a simplification of Postulate P_4 .

The simplification, it will be found, will retain all the advantages possessed by the older set. It consists merely in replacing P_4 by a proposition in which the negative elements are considerably fewer, and are more symmetrically distributed, than in P_4 .

In order to prove the sufficiency of the new postulates for Boolean algebras, it will of course suffice to show that my former set can be derived from them. This derivation I obtain. However, I also derive from the new postulates Sheffer's set and the well known Whitehead-Huntington set.§ Since the sufficiency of my former set is proved by showing that it yields Sheffer's set, and since the sufficiency of Sheffer's set is proved by showing that this set yields the Whitehead-Huntington set, I thought that it might be of interest to derive the latter two sets directly from my new set. These derivations, at the same time, will exhibit the workability of the new set.

* Presented to the Society, March 18, 1933.

† Transactions of this Society, vol. 17 (1916), pp. 50-52.

‡ See the Transactions of this Society, vol. 14 (1913), pp. 481-488.

§ See the Transactions of this Society, vol. 5 (1904), pp. 288-309.