INTEGRAL FUNCTIONS OBTAINED BY COMPOUNDING POLYNOMIALS*

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1. *Introduction.* We consider a sequence of polynomials $P_n(z)$, $(n=1, 2, \cdots)$, where the degrees of the P_n do not exceed a fixed integer m and where each P_n, ordered in ascending powers of z, starts with the term z. We shall study the sequence of polynomials $Q_n(z)$ defined by

$$(1) \qquad Q_1(z) = P_1(z);\ Q_{n+1}(z) = Q_n[P_{n+1}(z)], \qquad (n = 1, 2, \cdots),$$

and also the sequence of polynomials $R_n(z)$ defined by

$$(2) \qquad R_1(z) = P_1(z);\ R_{n+1}(z) = P_{n+1}[R_n(z)], \qquad (n = 1, 2, \cdots).$$

If the coefficients, after the first, in P_n, are sufficiently small, these sequences will converge to integral functions. For instance, sin z can be obtained, in many ways, as a limit of a sequence (1). In what follows, our chief object will be to establish conditions under which the sequences converge to integral functions.

2. *The Sequence of* $Q_n(z)$. Let

$$P_n(z) = z + a_{n2}z^2 + \cdots + a_{nm}z^m, \quad (n = 1, 2, \cdots),$$

where m is an integer independent of n.

THEOREM 1. *Let a convergent series of positive numbers,*

$$(3) \qquad c_1 + c_2 + \cdots + c_n + \cdots,$$

exist such that $|a_{ni}| < c_n$, *for every* n *and for* $i=2, \cdots, m$. *Then the sequence of polynomials* $Q_n(z)$ *converges to an integral function, the convergence being uniform in every bounded domain.*

PROOF. For every n,

$$(4) \qquad U_n(z) = z + c_n(z^2 + \cdots + z^m)$$

is a majorant of $P_n(z)$. Let

$$V_1 = U_1;\ V_{n+1} = V_n(U_{n+1}), \quad (n = 1, 2, \cdots).$$

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