There exists a metric space S in which Axioms 1, 3, 4, and 5′ hold true, but such that S is not completely separable. Hence it does not follow that a space S in which Axioms 1, 3, 4, and 5′ hold true is a subset of a plane, even if it is assumed that S is metric.

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ANALOGS OF THE STEINER SURFACE AND THEIR DOUBLE CURVES*

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The equations $x_1:x_2:x_3:x_4=x^n:y^n:z^n:w^n$, where x, y, z, w are linear functions of three homogeneous parameters, represent a rational surface of order n^2. For $n=2$ we have the well known Steiner surface. The particular subject of this paper is the double curve of such a surface and its representation on the plane. A few general properties must first be mentioned.

We take in the plane the reference system $x=0$, $y=0$, $z=0$, and $x+y+z\equiv -w=0$. The diagonals of the quadrilateral are $x+y\equiv -z-w=0$, etc. The vertices of the diagonal triangle are $(1:1:-1:-1)$, $(1:-1:1:-1)$, $(1:-1:-1:1)$, the fourth coordinate being w. Corresponding to the diagonals, the surface has 3 multiple right lines of order n, each meeting two opposite edges of the tetrahedron in points which correspond to a pair of opposite vertices of the quadrilateral. If n is even, the multiple lines are concurrent at $(1:1:1:1)$, which is a point of order $3(n-1)$ for the surface, corresponding to the vertices of the diagonal triangle and to certain pairs of imaginary points when $n>2$. If n is odd, the multiple lines are not concurrent, but are coplanar, meeting two by two at 3 points corresponding to the vertices of the diagonal triangle. The intersection of two multiple lines is then a point of order $2n-1$ for the surface. The class of the surface is always $3(n-1)^2$. The only pinch points are the 6 in which the multiple lines meet the edges of the tetrahedron. Each coordinate plane contains a single curve of order n, and is tangent to the surface along that curve, the order of contact being $n-1$. When n is even the section by a plane through a multiple line meets it in one variable real point, and

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