$$\int_{2\delta}^{a}\left|\log\left(1-\frac{\delta}{\tau}\right)\right|\frac{d\tau}{\tau} = -\int_{1/2}^{\delta/a}\left|\log(1-\tau)\right|\frac{d\tau}{\tau}$$
$$= \int_{\delta/a}^{1/2}\left|\log(1-\tau)\right|\frac{d\tau}{\tau} = o(1).$$

Hence $I_2 = o(1)$. Finally it is obvious that $I_3 = o(1)$ as $\delta \to 0$. On combining these facts, we obtain the desired result:

$$\int_{\delta}^{a}\left|\phi(t+\delta)-\phi(t)\right|\frac{dt}{t} = o(1).$$

BROWN UNIVERSITY

A NOTE ON COMPACTNESS*

BY E. H. HANSON

The purpose of this note is to deduce the conditions for compactness† of a set of measurable functions from the general criterion for compactness in complete metric spaces.‡ This is the procedure that suggests itself immediately and it suceeds without any difficulty. The general criterion referred to asserts that a set S of elements of a complete metric space is compact if and only if, for every positive ϵ, S is inclosable in a finite number of spheres§ of radius ϵ; and the validity of this the reader may easily verify for himself. Fréchet|| has obtained the result for

---

* Presented to the Society, February 25, 1933. I wish to express my gratitude to Professor Henry Blumberg for suggesting the idea of this note and for helpful criticism during its preparation.

† A set S of elements of a space is *compact* if every infinite subset of S has at least one limit point in the space.

‡ A space M is said to be *metric* if there exists a positive or zero function $d(e_1, e_2)$ of pairs of elements of M satisfying the conditions: (1) $d(e_1, e_2) = d(e_2, e_1)$, (2) $d(e_1, e_2) = 0$ is equivalent to $e_1 = e_2$; (3) $d(e_1, e_3) \leqq d(e_1, e_2) + d(e_2, e_3)$. A metric space is *complete* if $\lim_{m,n\to\infty} d(e_m, e_n) = 0$ implies the existence of an element e such that $\lim_{n\to\infty} d(e_n, e) = 0$.

§ A *sphere* with center c and radius r is by definition the set of elements e of M such that $d(e, c) < r$.

|| *Sur les ensembles compacts de fonctions mesurables*, Fundamenta Mathematicae, vol. 9, p. 25.