NOTE ON A SPECIAL CYCLIC SYSTEM*

BY MALCOLM FOSTER

1. *Introduction.* This note is concerned with a special cyclic system.† Let S be a surface referred to any orthogonal system, and T the trihedral whose x-axis is tangent to the curve $v=\text{const}$. The equations

$$(1)\qquad x=R(1+\cos\theta),\quad y=0,\quad z=R\sin\theta,$$

define a two-parameter family of circles C normal to S; and the necessary and sufficient conditions that C shall constitute a cyclic system are

$$(2)\ \xi\frac{\partial R}{\partial v}+R\eta_1 r=0,\quad R(pr_1-p_1r)-q_1\left(\xi+\frac{\partial R}{\partial u}\right)+q\frac{\partial R}{\partial v}=0.$$

It is readily seen that the first of equations (2) may be written‡

$$\xi\frac{\partial R}{\partial v}-R\frac{\partial\xi}{\partial z}=0;$$

hence

$$(3)\qquad R=U\xi,$$

where U is a function of u alone. Using (3) we may write the second equation of (2) in the form

$$(4)\quad U\xi(pr_1-p_1r)-q_1\left(\xi+\xi U'+U\frac{\partial\xi}{\partial u}\right)-qU\eta_1 r=0.$$

We shall therefore replace equations (2) by (3) and (4).

2. *The Inversion of C.* If we invert the circles C relative to the circles $x^2+z^2=K^2$, $y=0$, where K is any constant, we get the following system of lines L,

$$(5)\qquad x=\frac{K^2}{2R},\quad y=0,$$

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† See Eisenhart, *Differential Geometry of Curves and Surfaces*, Ex. 11, p. 444.

‡ Eisenhart, p. 170.