ON PRIMARY NORMAL DIVISION ALGEBRAS OF DEGREE EIGHT*

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1. *Introduction.* A normal division algebra A of degree n over F will be called *primary* if A is not expressible as a direct product of two normal division algebras B and C, where neither B nor C has degree unity. It is well known that necessarily $n=p^e$, p a prime, if A is primary. Moreover, if $n=p^e$, then a *sufficient* condition that A be primary is that A shall have exponent† n.

I have recently proved‡ that if A has degree four then A is primary if and only if A has exponent four. In the present paper I shall prove that there exist primary (cyclic) normal division algebras of degree eight but exponent four so that the above sufficient condition is actually not necessary.§

2. *Cyclic Fields of Degree Eight.*|| Let F be any non-modular field, and let C be a cyclic field of degree eight over F. Then I have proved that $C=F(x)$ contains a sub-field $F(y)$ which is cyclic of degree four over F and is defined by

$$y^2 = \nu(u-\tau),\ u^2 = \tau = 1+\epsilon^2, \tag{1}$$

where $\nu\neq 0$, $\epsilon\neq 0$ are in F and $\tau=1+\epsilon^2$ is not the square of any quantity of F. I have also proved that

$$-\nu\tau = \xi_1^2+\xi_2^2\tau,\ -\epsilon = (\eta_1^2-\eta_2^2\tau)(\xi_1^2+\xi_2^2\tau), \tag{2}$$

for ξ_1, ξ_2, η_1, η_2 in F. Conversely I have shown that if (1) and (2) are satisfied, then there exists a uniquely defined cyclic field $C=F(x)$ of degree eight over F and with $F(y)$ as cyclic quartic sub-field.

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† The exponent ρ of A of degree n is the least integer such that the direct power A^ρ is a total matric algebra, and is a divisor of n.

‡ Transactions of this Society, vol. 34 (1932), pp. 363–372.

§ It seems likely, however, that A of degree p^2, p a prime, is primary if and only if A has exponent p^2.

|| For proofs of the results of this section see my paper on cyclic fields of degree eight which has been offered for publication to the editors of the Transactions of this Society.