SOME INVOLUTORIAL LINE TRANSFORMATIONS*

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1. *Introduction.* In a paper published in this Bulletin,† the author discussed three involutorial line transformations obtained by considering the transversals of one line each of four plane pencils; one line each of two plane pencils and two generators of a quadratic regulus; and two generators each of two quadratic reguli. In this paper we shall discuss those involutorial transformations between the two transversals of three generators of a cubic regulus and one line of a plane pencil; and the transversals of four generators of a quartic regulus. We shall also discuss an involutorial line transformation defined by two plane harmonic homologies. The methods used are algebraic and the results obtained are interpreted as point transformations on a certain quadratic hypersurface in S_5.

2. *Cubic Regulus and Plane Pencil.* Consider the ruled surface

$$F_3 \equiv z_1^2 z_3 - z_2^2 z_4 = 0,$$

and the pencil of lines in the plane

$$\alpha \equiv z_1 + z_2 - z_3 + z_4 = 0,$$

with vertex $A \equiv (1, 0, 1, 0)$. The Plücker coordinates of a generator of F_3 and those of a line of the pencil (A, α) are $(0, -k^2, 1, 0, k, k^3)$ and $(1, 0, -m, m, 1+m, 1)$, respectively, where the k and m are parameters. A general line (y) with coordinates y_i, $(i=1, \cdots, 6)$, meets three generators of F_3 and one line of (A, α). These four lines, in general mutually skew, have a second transversal (x) whose co-ordinates are found to be

$$x_i = \phi_i(y), \qquad (i = 1, \cdots, 6), \tag{1}$$

where the $\phi_i(y)$ are cubic functions in y_j. Thus the transformation is of third order.

The invariant locus of the transformation is a quadratic complex whose equation is found in the manner described in the pre-

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