ON YOUNG'S DEFINITION OF AN ALGEBRA*

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1. *Introduction*. The usual definition of linear algebra† presupposes a number field over which the algebra is taken. Some of the properties of the algebra are dependent upon the properties of this associated field, for instance, the property that if e is an idempotent element, the subalgebra with basis e is a field and therefore permits unique division by every non-zero element.

Young‡ has given a definition of a general algebra and noted that a large part of the theory of linear algebras is valid for it. This definition does not introduce the associated field. Young concludes his paper with "Precisely to what extent, if at all, the present formulation is more general than the current one in terms of a field is a problem of no little interest." He notes the fact that every Dickson algebra is a Young algebra. That the converse of this is not true, however, is shown by the simple example of a modular ring. In fact, a modular ring possesses an idempotent element which generates by addition alone the entire algebra, yet division is not unique. Ingraham§ has noted this lack of restrictiveness of Young's definition, a lack of which, as I have been informed by Ingraham, Young had become cognizant.

The scalar multiplication in a Dickson algebra is not an operation within the algebra itself and it was with the view of eliminating this foreign operation that Young gave his definition. But, whether or not this operation is made a part of the definition of such an algebra, the possibility of defining an associated

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† L. E. Dickson, *Algebren und ihre Zahlentheorie*, Zurich, 1927, pp. 23–24 and p. 32. A linear algebra as defined by Dickson will hereafter be referred to as a *Dickson algebra*.

‡ J. W. Young, Annals of Mathematics, (2), vol. 29 (1927–28), pp. 54–60. A general algebra as defined by Young will hereafter be referred to as a *Young algebra*.

§ M. H. Ingraham, this Bulletin, vol. 38 (1932), p. 100.