QUADRATIC PARTITIONS: PAPER II

BY E. T. BELL

1. *Equivalent Identities.** We refer to I for the general notation, and now define a function F of 4 arguments by

$$F(w, z, u, v) \equiv F((z, u, v) \mid w) = -F(w, z, -v, -u);$$

that is, F is of the parity indicated and is arbitrary beyond the stated condition.

Obviously, $F(w, z, u, v)$ is an instance of the completely arbitrary parity function $f((z, u, v) \mid w)$ having parity $p(3 \mid 1)$, and $f((z, u, v) \mid w) - f((z, -v, -u) \mid w)$ is an instance of $F(w, z, u, v)$. Let $\sum_i$ refer to a set of values of (w, z, u, v). Then, by what precedes,

$$\sum_i c_i F(w_i, z_i, u_i, v_i) = 0,$$

$$\sum_i c_i [f((z_i, u_i, v_i) \mid w_i) - f((z_i, -v_i, -u_i) \mid w_i)] = 0$$

are both true or both false. If both are true, and hence if either is true, the relations are called *equivalent*. Many instances of equivalent identities will occur in these notes. The equivalence is always, as above, immediate from the properties of the functions implied in the parity notation, and need not be further discussed.

2. *ϑ, ϕ Identities.* Combined with the transformation of the second order, Jacobi's formula for the product of 4 ϑ-functions gives an endless chain of identities between functions ϑ, ϕ, which can be written down by elementary algebra.† These lead to identities between parity functions in any number of arguments summed over quadratic partitions. The first of these ϑ, ϕ identities to be discussed is

* The preceding note, this Bulletin, vol. 37 (1931), pp. 870–5, will be cited as I.

† At the writer's request, Mr. W. H. Gage of Victoria College, B.C., has derived a large number of these identities. A particular identity can easily be checked independently; Mr. Gage's systematic derivations reveal the simple interconnections between all. A sufficient account of the method will doubtless appear elsewhere.