NOTE ON THE LAW OF BIQUADRATIC RECIPROCITY*

BY V. W. ADKISSON

In outlining the proof of the law of biquadratic reciprocity H. J. S. Smith develops the expressions for S, T, S^4, and T^4† which are used in the proof given by Eisenstein.‡ We give here a slightly different development for these values making use of certain relationships established by Lebesgue.§ The advantage in this development lies in the fact that the function $\psi(i)$ is exhibited in a form which shows it to be a polynomial in i with integral coefficients, and that $\pm\psi(i)$ is a primary prime in the realm $k(i)$ if the proper sign be chosen. If

$$F(\alpha) = \sum_{n=0}^{p-2} \alpha^n x^{g^n},$$

where α is a root of the equation $(\alpha^{p-1}-1)/(\alpha-1)=0$, x is a root of $(x^p-1)/(x-1)=0$, g is a primitive root of p, and p is a prime of the form $4n+1$, then

$$F(\alpha)F(\alpha^{-1}) = \alpha^{(p-1)/2}p. \tag{1}$$

Substituting i for α, we obtain the result

$$F(i)F(i) = F(-1)\sum_{t=1}^{p-2} i^{\operatorname{ind} t}(-1)^{\operatorname{ind}(t+1)}.\| \tag{2}$$

Let

$$\psi(i) = \frac{[F(i)]^2}{F(-1)} = \sum_{t=1}^{p-2} i^{\operatorname{ind} t}(-1)^{\operatorname{ind}(t+1)}. \tag{3}$$

Hence, $\psi(i)$ is a polynomial in i with integral coefficients, and may be written in the form $a+bi$ where a and b are integers. But

---

* Research paper No. 280, University of Arkansas.

† H. J. S. Smith, *Collected Mathematical Papers*, vol. 1, pp. 78–87.

‡ E. Eisenstein, *Lois de réciprocité*, Journal für Mathematik (Crelle), vol. 28, pp. 57–67.

§ L. M. V.-A. Lebesgue, *Démonstration de quelques formules d'un mémoire de M. Jacobi*, Journal de Mathématiques (Liouville), vol. 19, pp. 289.

‖ Lebesgue, loc. cit.