cones projecting $C$ and $C^{\prime}$ from ( $0,0,1,0$ ) have contact of at least order $n+1$. Moreover, by changing this vertex to the point ( $a, b, 1,0$ ) it is easily shown by a method similar to that used in the general case that the cones projecting $C$ and $C^{\prime}$ from any point in the osculating plane have contact of order $n+1$. In other words, this special case arises when the principal plane coincides with the osculating plane.

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## ON RECTIFIABILITY IN METRIC SPACES

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1. Introduction. In Menger's studies in metrical geometry* considerable attention is given to the rectification of the simple arc and various definitions of the length of such an arc are discussed. With the definition of arc-length it is then possible to give conditions for the "Konvexifizierbarkeit" of a compact metric space ( p .96 ) and for the existence of a geodetic arc in a compact metric space (p. 492). Both theorems involve the assumption of the existence of a rectifiable arc between each pair of points. It is intended in this paper to show that these results and some others are due to space properties which are of a more general nature, at least formally, and which suggest possible further studies.
2. Intrinsic Distance. If $a$ and $b$ are two points of a metric space $Z$, we let $a b$ denote the distance between them. A finite set of points $\left\{a_{i}\right\}$ such that $a_{0}=a, a_{n}=b$, and every $a_{i} a_{i+1}<\delta$ will be called a $\delta$-chain from $a$ to $b$, and $a a_{1}+a_{1} a_{2}+\cdots+a_{n-1} b$ will be called its length. If we set $l_{\delta}(a, b)$ equal to the lower bound of the lengths of all $\delta$-chains from $a$ to $b$, it is clear that this number exists if there is any such chain, that it is greater than or equal to $a b$, and that it increases monotonely as $\delta \rightarrow 0$. The upper bound of $l_{\delta}(a, b)$ for all values of $\delta$ is called the intrinsic distance $\dagger$ from $a$ to $b$ and is denoted by $l(a, b)$.
[^0]
[^0]:    * Untersuchungen über allgemeine Metrik, Mathematische Annalen, vol. 100, pp. 75-163 and vol. 103, pp. 466-501. See also Annals of Mathematics, vol. 32, pp. 739-746.
    $\dagger$ This turns out to be essentially the same thing as Menger's "geodetic distance," loc. cit., p. 492 . See $\S \S 4$ and 7 below.

