TWO BOOKS ON DIFFERENTIAL GEOMETRY

Eléments de Géométrie Infinitésimale. By Gaston Julia. Paris, Gauthier-Villars, 1927. vi+242 pp.

Lehrbuch der Differentialgeometrie. By A. Duschek and W. Mayer. Band I, *Kurven und Flächen im Euklidischen Raum*, von A. Duschek. vi+250 pp. Band II, *Riemannsche Geometrie*, von W. Mayer. vi+245 pp. Leipzig and Berlin, B. G. Teubner, 1930.

It is interesting to compare Julia's book with the first volume of Duschek-Mayer's work, as they deal with the same subject, classical differential geometry. Both books use invariant notation; both endeavor to be accurate in the formulation of the theorems; both are of about one size. Still they differ greatly in general aspect and in material.

Julia's book follows more the classical pattern, and refers often to the treatises of Picard, Goursat, de la Vallée Poussin, Darboux, and to the work of Humbert. It uses vector methods for the formulation of certain general theorems, but often slips into coordinate notation, as required in special problems.

Duschek's book, on the contrary, persistently attempts to use not only vector methods, but also tensor calculus in ordinary differential geometry. As such, it is a pioneer work, with the possible exception of Ricci's never printed *Lezioni sulla Teoria delle Superficie* and J. E. Campbell's *Course of Differential Geometry* (1926), which, however, differ very much from Duschek's treatise. This results in a tendency to dwell upon theorems of a general nature.

A striking difference lies in the large amount of space that Julia devotes to the theory of contact and to the theory of envelopes (pp. 9–72, one-fourth of the book). Duschek devotes to this subject only a short discussion. Is it because Julia is in first instance an analyst? The modern theory of contact and envelopes was indeed introduced into differential geometry by two analysts, Lagrange and Cauchy. This fact makes Julia's book one of the best sources of information on contact and on envelopes. We find here contact of plane curves, of space curves, of curves and surfaces, and of surfaces. Then we have the discussion of the envelopes of systems of

(a) plane curves of equation $f(x, y, \alpha)=0$;
(b) plane curves of equation $f(x, y, \alpha, \beta)=0, \ \phi(\alpha, \beta)=0$;
(c) surfaces of equation $f(x, y, z, \alpha)=0$;
(d) surfaces of equation $f(x, y, z, \alpha, \beta)=0, \ \phi(\alpha, \beta)=0$;
(e) surfaces of equation $f(x, y, z, \alpha, \beta)=0$;
(f) space curves of equation $f(x, y, z, \alpha)=0, \ g(x, y, z, \alpha)=0$;
(g) space curves of equation $f(x, y, z, \alpha, \beta)=0, \ g(x, y, z, \alpha, \beta)=0$;

which discussion leads to the theory of congruences of curves, with mention of focal properties.

We should have liked to see here a more detailed discussion of the behavior of the envelope, taking higher derivatives into account. Apart from the paper by Risley and McDonald in the Annals of Mathematics of 1910–11 (second