THE CALCULUS OF VARIATIONS AND THE QUANTUM THEORY*

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1. Introduction. Several years ago an insistent curiosity caused me to forsake temporarily fields of mathematical interest more legitimate for me in order to find out if possible the character of the new quantum theories in which my physicist neighbors and some of my mathematical friends seemed to find so much of scientific interest and excitement. I was entirely ignorant of the theory at that time and was greatly surprised to find that my own specialty, the calculus of variations, through the media of mechanical and optical conceptions, had played an important role in its development. The quantum theory was then in an exceedingly fluent stage, and though the unanswered mathematical questions which presented themselves were numerous and significant it seemed to me useless for mathematicians to spend large amounts of time in endeavoring to perfect mathematical details when some new paper might at short notice cause fundamental changes in the whole structure of the theory. Every one who has followed the development of the quantum theory would agree, I think, that this impression was justified. At the present time the situation seems somewhat different. I have heard assurances from various reliable sources that the leaders in the promotion of quantum mechanics, men of wide physical experience and of daring in mathematical exploration distinctly beyond that of the average pure mathematician who takes his rules so seriously, have reached something like agreement. These statements should perhaps not be accepted too readily, as they sound like others that have been made in the past and which have afterward needed modification. But to the seeker after knowledge, less experienced in this field like myself, it also seems that an equilibrium has approx-

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