ON A COVARIANT DIFFERENTIATION PROCESS*

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1. *Introduction.* If the components of a tensor T^α_β are point functions, a tensor of one higher covariant order may be formed by differentiation and elimination. Specifically, the equations which express the law of transformation of the tensor are differentiated once with respect to each of the coordinate variables of a given set, and certain second derivatives appearing are eliminated by means of a relationship known as the fundamental affine connection. This process is called covariant differentiation and is one of the cardinal operations of the tensor calculus.†

It is the purpose of this note to point out that a somewhat similar process exist sfor tensors whose components are functions of x, dx/dt, d^2x/dt^2.

2. *Notation.* We shall suppose that the curves involved in the following discussions are given in parametric form and shall employ primes to indicate differentiation with respect to the parameter. Partial derivatives, for the most part, will be denoted by means of subscripts. Thus, we shall write x', x'', $\left\{{\alpha \atop \beta}\right\}_{x'^\gamma}$ for dx/dt, d^2x/dt^2, $\partial\left\{{\alpha \atop \beta}\right\}/\partial x'^\gamma$ respectively.

3. *The Differentiation Process.* We shall illustrate this process by applying it to a mixed tensor of the second order. Accordingly, suppose that the quantities $T^\alpha_\beta(x, x', x'')$ are defined along a given regular curve and are the components of a tensor of the type indicated. The extended point transformation,

$$
\begin{aligned}
&x^\alpha = x^\alpha(y^1, y^2, \cdots, y^n), \qquad x'^\alpha = \frac{\partial x^\alpha}{\partial y^i}\, y'^i, \\
&x''^\alpha = \frac{\partial x^\alpha}{\partial y^i}\, y''^i + \frac{\partial^2 x^\alpha}{\partial y^i \partial y^j}\, y'^i y'^j,
\end{aligned}
\tag{1}
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† For an exposition of tensor analysis and covariant differentiation in particular reference may be made to Oswald Veblen, *The Invariants of Quadratic Differential Forms*, 1927, Chapters 2, 3; L. P. Eisenhart, *Riemannian Geometry*, 1926, Chapter 1.