NOTE ON FUNCTIONAL FORMS QUADRATIC IN A FUNCTION AND ITS FIRST p DERIVATIVES*

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It has been shown that a normal functional form quadratic in a function and its derivative is reducible to an invariantive form quadratic in a function and a constant, and that the continuity of the coefficients of the first form implies the continuity of those of the second.† The result is here generalized to a form quadratic in a function and p derivatives.

Let $y_0{}^\alpha$ denote a continuous function of the real variable α defined on the range $a \leqq \alpha \leqq b$. Let $y_1{}^\alpha, y_2{}^\alpha, \cdots, y_p{}^\alpha$ denote the corresponding derivatives defined and continuous on the same interval. The normal quadratic form is

$$Q = \sum_{i,j=0}^{p} A_{\alpha\beta}^{ij} \overset{\alpha}{y_i} \overset{\beta}{y_j} + A_\alpha^{ij} \overset{\alpha}{y_i} \overset{\alpha}{y_j}, \tag{A}$$

where $A_\alpha{}^i{}_\beta{}^j$ and $A_\alpha{}^{ij}$ are functions integrable in the Riemann sense, and where a Greek index repeated as a subscript and as a superscript is understood to denote integration with respect to that variable over the fundamental interval (a,b). The form Q is such that no generality is lost by assuming that $A_\alpha{}^i{}_\beta{}^j = A_\beta{}^j{}_\alpha{}^i$ and $A_\alpha{}^{ij} = A_\alpha{}^{ji}$. This assumption is made.

The convention of denoting integration by repeated Greek indices will be used in all that follows. In addition, when a Latin index (except p) occurs in a single term once as a superscript and once as a subscript, we shall understand summation with respect to that index over an integer range $(0, p-1)$. Thus

$$\begin{aligned} Q = {} & A_{\alpha\beta}^{ij} \overset{\alpha}{y_i} \overset{\beta}{y_j} + A_{\alpha\beta}^{pj} \overset{\alpha}{y_p} \overset{\beta}{y_j} + A_{\alpha\beta}^{ip} \overset{\alpha}{y_i} \overset{\beta}{y_p} + A_{\alpha\beta}^{pp} \overset{\alpha}{y_p} \overset{\beta}{y_p} \\ & + A_\alpha^{ij} \overset{\alpha}{y_i} \overset{\alpha}{y_j} + A_\alpha^{pj} \overset{\alpha}{y_p} \overset{\alpha}{y_j} + A_\alpha^{ip} \overset{\alpha}{y_i} \overset{\alpha}{y_p} + A_\alpha^{pp} (\overset{\alpha}{y_p})^2. \end{aligned} \tag{A'}$$

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† A. D. Michal and L. S. Kennison, *Quadratic functional forms on a composite range*, Proceedings of the National Academy of Sciences, vol. 16 (1930), pp. 617–619.