From the above and from (8), we obtain

$$\left\{\frac{1-\alpha}{r}\right\} = \left\{\frac{\alpha}{r}\right\},$$

and from (7) it follows that

$$r^{p-1} \equiv 1 \pmod{p^2}.$$

As before, a similar proof obtains when y is divisible by p.

OTTAWA, CANADA

ON THE SOLUTION OF THE EULER EQUATIONS FOR THEIR HIGHEST DERIVATIVES*

BY H. V. CRAIG

1. *Introduction.* J. H. Taylor† has given two elegant methods of solving for their highest derivatives the Euler equations associated with the integral $\int F(x, \dot{x})dt$. In this paper these two methods are modified so as to apply to the more general case in which the Euler equations contain derivatives of order higher than the second.

2. *Notation.* Throughout this paper we shall employ vector notation and shall use dots and enclosed superscripts to indicate differentiation with respect to the parameter. Thus x, $\dot{x}$, $x^{(m)}$ will stand for the sets

$$x^1, x^2, \cdots, x^n; \frac{dx'}{dt}, \frac{dx^2}{dt}, \cdots, \frac{dx^n}{dt}; \frac{d^m x^1}{dt^m}, \frac{d^m x^2}{dt^m}, \cdots, \frac{d^m x^n}{dt^m},$$

respectively. Partial derivatives will be denoted by means of subscripts, thus

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† J. H. Taylor, *The reduction of Euler's equations to a canonical form*, this Bulletin, vol. 31 (1925) p. 257.