So, for the same value of h, namely, $\bar{h}$, there are two values of θ, namely, $\bar{\theta}$ and $\bar{\theta}'$, which is absurd, since θ is single-valued.

(b) If the discontinuity is of the second kind, then there must be a sequence $\{h_n\}$, tending to $\bar{h}$, for which the corresponding sequence $\{\xi_n\}$ does not tend to any limit. Therefore two values k_1 and k_2 of h can always be found as near as we please to $\bar{h}$ such that the corresponding values η_1 and η_2 of ξ differ from each other by a quantity greater than a suitably prescribed positive quantity δ. But, from (M), $f(h)/h$ and, consequently, $f'(\xi)$ are continuous functions of h at $\bar{h}$. Therefore ξ must be multiple-valued at $\bar{h}$, which is absurd, since θ is single-valued.

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A NUMERICAL FUNCTION APPLIED TO CYCLOTOMY

BY EMMA T. LEHMER

A function $\phi_2(n)$ giving the number of pairs of consecutive integers each less than n and prime to n, was considered first by Schemmel.* In applying this function to the enumeration of magic squares, D. N. Lehmer† has shown that if one replaces consecutive pairs by pairs of integers having a fixed difference λ prime to $n=\prod_{i=1}^{t} p_i^{\alpha_i}$, then the number of such pairs (mod n) whose elements are both prime to n is also given by

$$\phi_2(n)=\prod_{i=1}^{t} p_i^{\alpha_i-1}(p_i - 2) .$$

As is the case for Euler's totient function $\phi(n)$, the function $\phi_2(n)$ obviously enjoys the multiplicative property $\phi_2(m)\phi_2(n) =\phi_2(mn)$, $(m, n)=1$, $\phi_2(1)=1$. *In what follows we call an integer simple if it contains no square factor* >1. For a simple number n we have the following analog of Gauss' theorem:

$$\sum_{\delta\mid n}\phi_2(\delta) = \phi(n), \tag{1}$$

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* Journal für Mathematik, vol. 70 (1869), pp. 191–2.

† Transactions of this Society, vol. 31 (1929), pp. 538–9.