ON THE NATURE OF θ IN THE MEAN-VALUE THEOREM OF THE DIFFERENTIAL CALCULUS*

BY GANESH PRASAD

1. *Introduction.* If $f(x)$ is a single-valued function which is finite and continuous in an interval (a, b), the ends being included, than the relation

$$\text{(M)} \qquad f(x+h) = f(x) + hf'(x+\theta h), \qquad 0 < \theta < 1,$$

holds for every value of x and h for which the interval $(x, x+h)$ is in the interval (a, b); provided that *either* $f'(x)$ exists at every point inside the interval (a, b) *or* a certain less restrictive condition† is satisfied. In recent years the nature of θ has been studied by a number of writers‡ who start with the assumption that $f''(x)$ exists everywhere in the interval (a, b). The two theorems, which it is the object of this paper to formulate and prove, are believed to be new and hold even if $f''(x)$ does not exist everywhere. For the sake of clarity and fixity of ideas, I consider θ only as a function of h, assuming x to be a constant, say 0, in the theorem (M).

2. THEOREM I. *If $\theta(h)$ is single-valued and continuous, it is not necessarily differentiable for every value of h.*

PROOF. Take $f(x)$ to be the indefinite integral of a monotone, increasing and continuous function which has a differential coefficient everywhere in the interval (a, b), excepting the points

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† The condition of W. H. Young and G. C. Young, Quarterly Journal of Mathematics, vol. 40 (1909), p. 1; Hobson's *Theory of Functions of a Real Variable*, vol. 1, 3d edition, 1927, p. 384; or the still less restrictive condition of A. N. Singh, Bulletin of the Calcutta Mathematical Society, vol. 19 (1928), p. 43.

‡ R. Rothe (Mathematische Zeitschrift, vol. 9 (1921), p. 300; Tôhoku Mathematical Journal, vol. 29 (1928), p. 145); T. Hayashi (Science Reports of the Tôhoku Imperial University, (1), vol. 13 (1925), p. 385); O. Szász (Mathematische Zeitschrift, vol. 25 (1926), p. 116).