THE GENERAL THEORY OF FACTORIAL SERIES*

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1. *Introduction.* The subject of this paper is factorial series, a class of series which it seems to me are more interesting than American mathematicians generally realize and which can be made to play a most important part in analysis.

By a factorial series we shall understand a series of the form

$$c_0 + \sum_{n=1}^{\infty} c_n \frac{(n-1)!}{z(z+1)\cdots(z+n-1)} = \Omega(z), \tag{1}$$

where c_n is a sequence of constants and z a complex variable. The term is sometimes also made to include series in non-inverted factorials or the so-called binomial-coefficient series. I shall use the term only as applying to series (1) and at times to certain generalizations of which I shall speak presently. The terms of (1) are not defined when $z=0, -1, -2, \cdots$. Such points will be understood throughout the paper as excepted in all theorems, and neighborhoods of them in all theorems relating to uniformity.

We notice the simple form of series (1), and comparison with series of negative powers, $c_0+\sum_{n=1}^{\infty}c_n(n-1)!\, z^{-n}$, immediately is suggested. We observe first that the terms of the series of negative powers after the first, but for coefficient, come from successive differentiations of $1/z$. The terms of the factorial series come from taking successive differences of $1/z$. The power series yields itself readily to differentiation and integration and certainly a large part of its usefulness in analysis comes from this fact. The factorial series (1) is not so readily differentiated or integrated. However,

$$\Delta\Omega(z) = \Omega(z+1) - \Omega(z) = -\sum_{n=1}^{\infty} c_n \frac{n!}{z(z+1)\cdots(z+n)},$$

another factorial series of equally simple form, quite a parallel

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