A CORRESPONDENCE CONNECTED WITH A PENCIL OF CURVES OF ORDER n

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All curves of order n passing through $\frac{1}{2}n(n+3)-1$ points pass through $\frac{1}{2}(n-1)(n-2)$ other points. Or we may say that the n^2 base points of a pencil of curves of order n are determined by the number given above. When $\frac{1}{2}n(n+3)-2$ are fixed and another moves on a curve of order m, the locus of the remaining $\frac{1}{2}n(n-1)(n-2)$ is a curve of order $m(n^2-1)$ which has a multiple point of order mn at each of the fixed points. The order of the locus is reduced by n for each passage of the given curve through a fixed point.* It is the purpose of this paper to discuss the locus of the remaining base points of a pencil when a number of them are fixed and the others necessary to determine the pencil are taken consecutive on some curve.

We consider first the case when $\frac{1}{2}n(n+3)-3$ points are fixed. Thus for example taking $n=4$, we fix 11 base points of a pencil of quartics and let 2 be consecutive on a line l. To find the locus of the remaining three we use the rational quintic surface with a double cubic whose plane sections correspond to the ∞^3 quartics through the 11 fixed points. To l corresponds on the surface a rational twisted quartic C_4; and to a pencil of quartics through the 11 points and tangent to l corresponds a pencil of plane sections whose axis is tangent to C_4. This tangent meets the quintic surface in 3 more points which correspond to the three remaining base points of the pencil of plane quartics. The tangents to C_4 form a developable of order 6. Therefore, the plane curve which corresponds to the intersection of the developable with the quintic is of order 6×4 and has a sextuple point at each of the 11 fixed points. In this locus the line l is counted twice. Hence the locus sought is of order 22. In addition to the singularities at the fixed points

* Milinowski, Journal für Mathematik, vol. 67, p. 263. For $n=3$ we have the Geiser involutorial transformation of order 8.