ON ANALYTIC SOLUTIONS OF DIFFERENTIAL EQUATIONS IN THE NEIGHBORHOOD OF NON-ANALYTIC SINGULAR POINTS*

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We shall consider the system

$$\frac{dx_1}{X_1} = \frac{dx_2}{X_2} = \cdots = \frac{dx_n}{X_n} \tag{1}$$

in the complex neighborhood of a point, say $(0, 0, \cdots, 0)$, at which the X's vanish simultaneously. We shall suppose throughout that

$$X_i = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n + \phi_i(x_1, \cdots, x_n), \qquad (i = 1, 2, \cdots, n),$$

and that the general solutions of the related linear system

$$\frac{dx_1}{a_{11}x_1 + \cdots + a_{1n}x_n} = \cdots = \frac{dx_n}{a_{n1}x_1 + \cdots + a_{nn}x_n} \tag{2}$$

approach zero, without being identically zero.‡ Consider a solution of the equations (2). When will there exist an analytic solution of the equations (1) which behaves essentially like this near $(0, 0, \cdots, 0)$? How many such solutions will there be? Our object is to answer these questions without restricting the functions ϕ_i, except as regards their behavior in a certain region of the complex $x_1x_2 \cdots x_n$-space abutting on the origin, in which the given solution of (2) is embedded. We shall thus obtain a treatment less restricted than those of Briot and Bouquet, Poincaré, Picard, Dulac, Horn, etc., who assume the ϕ's to be convergent power series of terms of degree higher than the first.§

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‡ See, however, the concluding note of this paper.

§ For full references, see J. Malmquist, *Sur les points singuliers des équations différentielles*, Arkiv för Matematik, Astronomi och Fysik, vol. 15, No. 3, (1920), pp. 1–5.