SOME THEOREMS CONCERNING MEASURABLE FUNCTIONS*

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Theorems on the measurability of functions of measurable functions, e. g., in the form $F(x)=f[x, g(x)]$, have been given by Carathéodory and other writers.‡ Our Theorem I is an easy generalization of the one given by Carathéodory on page 665, with a slightly different method of proof. Here the function $f(x, y)$ is supposed to be defined for all values of y. Our Theorem II merely applies Theorem I to certain cases when the function $f(x, y)$ is *not* defined for all values of y. In these theorems the variables x and y may be multipartite. Theorems I and II are still valid if, throughout, *measurable* is replaced by *Borel measurable*.

In Theorem III, we consider a summable function $f(x, y)$ of two variables, and show by means of Theorem I that the function of x alone

$$\int_a^x f(x,y)dy$$

is also summable, under a suitable convention.

Notations. In Theorems I and II we use the following abbreviated notations: The point $(x_1, \cdots, x_k)$ in k-dimensional space, we denote simply by x. The x-space as a whole is denoted by the German $\mathfrak{X}$. We do similarly for the m-dimensional space $\mathfrak{Y}$. When we have to speak of the $(k+m)$-dimensional space $(\mathfrak{X}, \mathfrak{Y})$, we may denote

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‡ See Carathéodory, *Vorlesungen über reelle Funktionen*, pp. 376,377,665; Hans Hahn, *Theorie der reellen Funktionen*, p. 556.
Hobson, *Theory of Functions of a Real Variable*, 2d ed., vol. 1, p. 518.