ON THE SERIAL RELATION IN BOOLEAN ALGEBRAS*

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The elements $x, y, z, \cdots$ of a class K are said to form a *series* with respect to a dyadic relation S, if they satisfy the following postulates:†

P_1. If $x \neq y$, then either xSy or ySx.

P_2. If xSy, then $x \neq y$.

P_3. If xSy and ySz, then xSz.

It is my object to determine all serial relations in Boolean algebras given by universal propositions expressible in the fundamental Boolean operations of addition, multiplication, and negation.

All the desired serial relations S must be of the form

$$axy + bxy' + cx'y + dx'y' = 0, \tag{1}$$

where x' is the negative of x. Our problem then reduces itself to finding the conditions imposed on the coefficients of (1) by P_1-P_3. We proceed to determine these conditions.

If in (1) $x=0$ and $y=1$, then $c=0$; if $x=1$ and $y=0$, then $b=0$. Hence the condition imposed on (1) by P_1 is

$$b = 0 \quad \text{or} \quad c = 0. \tag{2}$$

The condition imposed on (1) by P_2 is that the equation $axx + bxx' + cx'x + dx'x' = 0$ have no solution. This condition is

$$ad \neq 0. \tag{3}$$

The condition that (1) satisfies P_3 non-vacuously is (this BULLETIN, vol. 30, p. 127)

$$a + d < b + c, \quad ad = 0, \tag{4}$$

which contradicts (3). Hence, *there are no non-vacuous serial relations* (1) *in any Boolean algebra.*

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† See E. V. Huntington, *The Continuum*, 2d ed., p. 10.