SHORTER NOTICES

Elementi del Calcolo delle Variazioni. By G. Vivanti. Bibliotheca di mate-
   matiche superiori diretta da Roberto Marcolongo e Gaetano Scorza.
   Messina, Giuseppe Principato, 1923. 296 pp.

One of the most interesting features of this book is the large number of
examples in the calculus of variations which are discussed throughout the
text. Theoretical questions have not been neglected, but after treating
them concisely the author hastens to elucidate his theory by applying it
to numerous illustrations. He has followed the example set by Kneser in
listing his problems by number at the end of the book, with indications
of the pages on which they are considered. There are twenty-four in all.
Most of them are well known, but one may be especially mentioned. It is
an isoperimetric problem (No. 14, p. 93) whose extremals are straight lines,
which has only solutions with corners, and whose conjugate points are
determined by a simple geometric construction.

Mathematicians value highly the books on algebraic geometry which
are devoted to the theory of special algebraic curves or surfaces. It has
always seemed to me that for similar reasons a book devoted entirely to
the elucidation of special problems of the calculus of variations, in which
the theory of each problem would be developed consecutively as far as it
is known, would be of great value. The text of Professor Vivanti would
give important assistance in preparing such a treatise.

The book before us is divided into two principal parts, devoted, respec-
tively, to the conditions arising from the first and second variations, and
there is an appendix on functional calculus and its applications to the proof
of the existence of absolute minima. The variety of problems of the cal-
culus of variations treated is large, the more important ones being the
simplest problem in the plane, parametric problems, problems whose
solutions have corners or arcs in common with the boundary of a region,
isoperimetric problems, problems with variable end-points, problems of the
Lagrange type, and those involving double integrals. One chapter in each
part is devoted to each of these cases. The simplest problem in the plane
in non-parametric and parametric forms is treated with considerable
completeness, but the conditions deduced for the other problems are for
the most part necessary conditions. In a number of cases references are
given to Bolza for sufficiency proofs, and in others the proofs are omitted
or passed over somewhat hastily.

I should like to call attention here to the method which I have used for
deducing the necessary condition of Jacobi from the second variation.* It
is widely applicable and relatively simple. In parametric cases particularly

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* TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916), p. 195; Bliss, *Calculus
of Variations*, p. 161; Smith, TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916),
p. 459; Larew, *ibid.*, vol. 20 (1919), p. 1.