NOTE ON A THEOREM OF KEMPNER CONCERNING TRANSCENDENTAL NUMBERS*

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The theorem in question is the following one:†

Let a be an integer greater than 1, p/q *a rational fraction,* $p \gtrless 0$, $q>0$; $\alpha_n(n=0, 1, 2, \cdots)$, *any positive or negative integer smaller in absolute value than a fixed arbitrary number* M, *but only a finite number of the* α_n *equal to* 0; *then*

$$\sum_{n=0}^{\infty} \frac{\alpha_n}{a^{2^n}}\left(\frac{p}{q}\right)^n$$

is a transcendental number.

Professor Kempner states: "The condition that only a finite number of coefficients shall be zero $\cdots$ I have not been able to remove."

Now although the proof of the theorem appears essentially to depend not merely on the *croissance* of the denominators a^{2^n} but also on the particular character of the exponent 2^n of a, so that considerations based on the representation of numbers in the binary scale may be used, it nevertheless seems plausible that the restriction that only a finite number of coefficients shall be zero is dispensable. And, indeed, it is the purpose of this note to prove the theorem without this restriction; in other words, to prove the following theorem.

THEOREM. *The properly‡ infinite series*

$$f(x) = \sum_{n=0}^{\infty} \frac{\alpha_n}{a^{2^n}} x^n ,$$

where a is an integer greater than 1, *and* α_n *an integer less in absolute value than a fixed number* M, *is transcendental for rational* $x(\neq 0)$.

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† TRANSACTIONS OF THIS SOCIETY, vol. 17 (1916), p. 477.

‡ That is, the terms are not all zero after a certain point.