A NOTION OF UNIFORM INTEGRABILITY*

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The necessary and sufficient condition that a function $f(x)$ of the real variable x be integrable in the sense of Riemann on the interval (a,b) is that there correspond to an arbitrary small positive number ϵ a positive δ such that for any subdivision of (a,b) by points

$$x_0=a\leqq x_1\leqq x_2\leqq \cdots \leqq x_{n-1}\leqq x_n=b\ ,$$

subject to the condition $x_i-x_{i-1}<\delta$, the inequality

$$\sum_{i=1}^{n}(U_i-L_i)(x_i-x_{i-1})\leqq\epsilon$$

is valid. In this, U_i and L_i represent, respectively, the upper and lower bounds of $f(x)$ on the subinterval (x_{i-1},x_i).

In the direct extension of this definition to a function which involves besides the variable of integration also other parameters, it may or may not be possible in any particular case to satisfy the conditions above by a constant δ independent of the parameters. In this connection the following concept may be of interest.

A function $f(x,\lambda)$ shall be defined to be integrable with respect to x on (a,b) uniformly in λ, provided that there corresponds to an arbitrary positive ϵ a positive constant δ independent of λ, such that

$$(1)\qquad \sum_{i=1}^{n}\left\{U_i(\lambda)-L_i(\lambda)\right\}(x_i-x_{i-1})\leqq\epsilon.\dagger$$

If $f(x,\lambda)$ is complex, it shall be said to be uniformly integrable if both its real and its imaginary parts are uniformly integrable.

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† The extension of this definition to the case when f involves a greater number of parameters, real or complex, is, of course, immediate.