# THE ALTERNATION OF NODES OF LINEARLY INDEPENDENT SOLUTIONS OF SECOND ORDER DIFFERENCE EQUATIONS*

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We shall consider solutions of the difference equation

$$(1) \qquad u(n+2) = A(n)\, u(n+1) - B(n)\, u(n), \qquad B(n) > 0,$$

where $A(n)$ and $B(n)$ are finite and single-valued functions of the integer $n$. If the points obtained by plotting a solution $u(n)$ are joined by segments of a straight line, this broken line gives a representation of a single-valued and continuous function $f(x)$ such that $f(n) = u(n)$. The zeros of $f(x)$ are called the nodes of $u(n)$.

Proofs have already been given of the following theorem.

THEOREM. *The nodes of two linearly independent solutions of* (1) *separate one another.*†

The proof which is to be given here seems simpler and more obvious than either of these two proofs. Two known and easily verified facts will be used. If $u_1(n)$ and $u_2(n)$ are any two solutions of (1) and if we set

$$\Delta(n) = \begin{vmatrix} u_1(n) & u_2(n) \\ u_1(n+1) & u_2(n+1) \end{vmatrix},$$

then

$$\Delta(n+1) = B(n)\Delta(n) \ .$$

As a first result of the condition imposed upon $B(n)$ in (1) we have the fact that if $\Delta(n)$ is not zero for one value of $n$ then it is never zero and its sign remains unchanged.

A necessary and sufficient condition that the two solutions $u_1(n)$ and $u_2(n)$ are linearly independent is that $\Delta(n)$ is not zero for one value of $n$.

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† Porter, ANNALS OF MATHEMATICS, (2), vol. 3, (1901–02), p. 65. Moulton, E. J., ibid. ,(2), vol. 13 (1911–12), p. 137.