NOTES ON THE RATIONAL PLANE CUBIC CURVE*

BY T. W. MOORE

1. *Involutions.* The rational cubic curve in the plane can be defined parametrically by three equations of the type

$$x_i = (\alpha_i t)^3\,, \qquad (i=1,2,3)\,. \tag{1}$$

Then any line $(\xi x)=0$ will determine the binary cubic

$$(a\xi)(\alpha t)^3 = 0\,, \tag{2}$$

on the curve and we have an involution of line sections. There is a unique binary cubic $(\beta t)^3=0$, called the fundamental cubic of the R_2^3, apolar to each of the three cubics (1) and hence apolar to any line section. The condition that any three points of the base cubic R_2^3 be on a line is that their parameters satisfy the relation

$$\beta_0 s_3 + \beta_1 s_2 + \beta_2 s_1 + \beta_3 = 0\,,$$

the polarized form of $(\beta t)^3=3$. The roots of $(\beta t)^3=0$ are the triple points of the involution of line sections and are thus the parameters of the flexes.

If we denote by $(\gamma t)^3$ the cubic covariant of $(\beta t)^3$, then the polarized form of $(\gamma t)^3=0$ represents another cubic involution $I_{2,1}^2$, on the base curve, of which the roots of the cubic covariant are triple points. These three points are the sextactic points, contacts of the tangents from the flexes, and consequently the intersections of the harmonic polars with the curve. Winger† has shown that this involution is determined by the contacts of the tri-tangent conics. The tri-tangent conics are the envelopes of lines joining points apolar to given fixed pairs of points of the base cubic. But what is more important here is the fact that this involution of points is also cut out of the cubic by the net of conics on the three sextactic points. We can see

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* Presented to the Society, May 1, 1926.

† R. M. Winger, this BULLETIN, vol. 25 (1918–19), pp. 27–34.