AN APPROXIMATION TO THE LEAST ROOT OF A CUBIC EQUATION WITH APPLICATION TO THE DETERMINATION OF UNITS IN PURE CUBIC FIELDS

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1. *Approximation to the Least Root of a Cubic.* Bernoulli's method of approximating the largest root of an equation

$$x^3 = ax^2 + bx + c, \tag{1}$$

with real coefficients, is to use (1) as a scale of relation for the recursion formula $A_n = aA_{n-1} + bA_{n-2} + cA_{n-3}$. Successive A's are calculated starting from any initial values. Then A_{n+1}/A_n for increasing values of n approximates that root of (1) which has the greatest absolute value if that root is real. The method here given for approximating the least root of (1) is similar to Bernoulli's. We use three recursion formulas

$$\begin{cases} A_n = aA_{n-1} + bA_{n-2} + cA_{n-3}\ , \\ B_n = aB_{n-1} + bB_{n-2} + cB_{n-3}\ , \\ C_n = aC_{n-1} + bC_{n-2} + cC_{n-3}\ , \end{cases} \tag{2}$$

and calculate the successive A's, B's, and C's starting from the three sets of initial values

$$\begin{cases} (A_{-2}\ , A_{-1}\ , A_0) = (0,0,1)\ , \\ (B_{-2}, B_{-1}, B_0) = (0,1,0)\ , \\ (C_{-2}, C_{-1}, C_0) = (1,0,0)\ . \end{cases} \tag{3}$$

Then the quotient

$$-\frac{\begin{vmatrix} A_n & C_n \\ A_{n+1} & C_{n+1} \end{vmatrix}}{\begin{vmatrix} A_n & B_n \\ A_{n+1} & B_{n+1} \end{vmatrix}}\ , \tag{4}$$