SOME MODERN VIEWS OF SPACE*

BY JAMES PIERPONT

1. Introduction. We are living in an age of great discoveries; in physics, in chemistry, in astronomy; in the field of invention one is almost bewildered by the great achievements which have been made in recent years. Although figuring less in the public eye, the development of mathematics has been no less remarkable. In the present paper I wish to outline briefly what progress has been made in our knowledge of space from a mathematical standpoint.

Until about a century ago everybody believed that the geometry of Euclid gave an exact description of space as far as it went. Geometry as the science of space has to deal with points, straight lines and planes. What are these things? Euclid says: A point is that which has no part, a line is breadthless length, a straight line is a line which lies evenly with the points on itself, and so on. I do not need to continue. As we see, these definitions would not tell one what a point, a straight line, a plane, are if one did not already have these notions in his mind. Euclid probably did not intend that they should be regarded otherwise than briefly describing some of their salient properties.

We see on all sides of us lines which are approximately straight, and surfaces which are approximately plane. A stretched string or a ray of light visualize a straight line, and the surface of a pond a plane. In machinery plane surfaces are of great importance; the engineer, the physicist and the astronomer are vitally interested in them. How are they constructed? One takes three metal plates nearly plane and rubs them pairwise together using some abrasive powder. In this way we get slightly spherical surfaces, one concave and two convex or two concave and one convex.

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