ON THE SOLUTION OF HIGHER DEGREE ALGEBRAIC EQUATIONS*

BY GLENN JAMES

1. *Introduction.* In this paper, we first solve for a real root of the general algebraic equation with real coefficients and negative constant term. This root appears as the limit of a function defined by a certain recursion formula. Ordinary radicals are special forms of it. By means of this result and the notation of repeated resolvent equations, we then outline a theoretically possible method of obtaining formulas for all the roots of any equation.

2. *The Least Positive Real Root.* Consider the equation

$$x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n=0\,, \tag{1}$$

where the coefficients are real and $a_n<0$. At least one root of this equation lies between 0 and k where

$$k>|a_1|+\sqrt{|a_2|}+\sqrt[3]{|a_3|}+\cdots+\sqrt[n]{|a_n|}\,,$$

and no root as large as k.† In order to simplify our work and our results, we subtract k from the roots of (1) then make use of the interval $-k$ to 0. The new equation is

$$(x+k)^n+a_1(x+k)^{n-1}+\cdots+a_n=0\,. \tag{2}$$

We now denote the left member of this equation by $f(x)$ and make use of the interpolation formula

$$\frac{x_p}{x_{p-1}}=\frac{f(0)}{f(0)-f(x_{p-1})}\,,\qquad p=2\,,3\,,\cdots\,,\quad x_1=-k. \tag{3}$$

---

* Presented to the Society, San Francisco Section, April 4, 1925.

† J. L. Walsh, (ANNALS OF MATHEMATICS, (2), vol. 25, No. 3, p. 285) proves that in the complex plane the roots all lie within or on a circle about the origin having this expression for a radius. This can be proved for real values by direct substitution.