ON IRREDUNDANT SETS OF POSTULATES*

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In his paper *On irredundant sets of postulates*,† Mr. Alonzo Church gives a *mechanical method*‡ by which any set of postulates can be made irredundant. This method in the general case is as follows. Given a set of postulates $A_1, A_2, \cdots, A_n$. Form the set of postulates $B_1, B_2, \cdots, B_n$, where $B_1=A_1$ and for each $i(i=2, 3, \cdots, n)$, B_i denotes the proposition *if* $A_1, A_2, \cdots, A_{i-1}$, *then* A_i.

Obviously the negatives of any two postulates of the set [B] are contradictory. Hence to show that the set [B] is irredundant, we need merely show the postulates independent by showing for each $i(i=1, 2, \cdots, n)$, an example in which B_i is false. This requires the existence of examples exhibiting these characteristics in terms of the set [A]: $A_1, A_2, \cdots, A_{i-1}$ true, A_i false, for each i.

Even if the postulates of set [A] are not independent, the postulates of set [B] are independent (and irredundant), except when a relation exists of this form:

(I) If $A_{n_1}, A_{n_2}, \cdots, A_{n_{k-1}}$, then A_{n_k},
$$\text{for } 1 \leqq n_1 < n_2 < \cdots < n_{k-1} < n_k \leqq n,$$

in which case the postulates of set [B] are not independent.

We have here a new method of obtaining independence among postulates. *Given any set of n postulates* [A] *which can be arranged in a sequence such that no relation of form* (I) *exists. The set* [A] *can be replaced, without losing any implications, by*

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† TRANSACTIONS OF THIS SOCIETY, vol. 27 (1925), p. 318. A set of postulates is *irredundant* if the postulates are independent and the negatives of every two are contradictory.

‡ Loc. cit., p. 321. Church confines his remarks to the case where the postulates are independent.