PROPERTIES OF UNRESTRICTED REAL FUNCTIONS*

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The functions that ordinarily interest mathematicians are of specialized character—continuous, differentiable, analytic, of limited variation, etc. Historically, of course, the race starts with simple, concrete things and only gradually moves on to abstract conceptions. In the case of functions, it was not till the nineteenth century that serious attention was accorded functions affected with a generous degree of discontinuity, and it was not till the middle of the century that there emerged Dirichlet's conception of an unrestricted (real) function. According to this conception, $g(x)$ is a real function of the real variable x if to every real number x there corresponds a real number $g(x)$. This conception, natural and simple though it is, conflicted with the traditional notion —which, indeed, is the same as the one widely held by those unconversant with modern developments of the theory of functions of a real variable—that required from every function some sort of analytic expressibility. Thus, $g(x)$ is a function, according to Dirichlet, if $g(0)=0$ and $g(x)=1$ for $x\neq 0$. It so happens that this particular $g(x)$ is analytically expressible as

$$\lim_{n\to\infty} \left(x^{\frac{1}{2n+1}}\right)^2$$

for example, where n is a positive integer. But the decision that $g(x)$ is a function rests, for Dirichlet, solely on the ground of the correspondence of a real number $g(x)$ to every real number x; whereas, according to the older conception,

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