COOLIDGE ON PROBABILITY

An Introduction to Mathematical Probability. By Julian L. Coolidge. Oxford, Clarendon Press, 1925. xi+215 pp.

This book meets a real need for a textbook on probability written in English and giving an introduction to a wide range of topics. The work is based on lectures by Professor Coolidge given at Harvard University. The purpose expressed in the preface is to give the mathematical basis underlying each of the important applications of probability rather than to write a treatise on games of chance, or errors of observations, or statistics, or statistical mechanics, or insurance. In the opinion of the reviewer, the author has succeeded well in carrying out his purpose in brief space.

The first chapter deals with the meaning of probability. Various definitions of probability are described. Then by means of three broad empirical assumptions the statistical definition of probability is given and adhered to throughout the book. On the whole, a good case seems to be made for the underlying rationale adopted. Moreover, the definition adopted surely lends itself to important statistical applications. The elementary principles for the combination of probabilities and the meanings of expectation and risk are clearly developed in the second chapter. The third chapter deals with the Bernoulli theorem which is rather commonly regarded as the central theorem underlying the main applications of probability in practical statistics. A very commendable feature of this chapter is the clearness and emphasis with which approximate results are set down as approximations. For example, it is emphasized that the Gaussian probability integral is merely an approximation to the probability, in repeated trials, that the discrepancy is numerically equal to or less than an assigned value. Hence, it is properly called a "fortunate accident" that

$$\frac{2}{\pi}\int_0^\infty e^{-z^2}\,dz=1\,,$$

and it is another fortunate accident that

$$\frac{1}{\sqrt{2\pi npq}}\int_0^\infty x^2e^{-\frac{x^2}{2npq}}\,dx=npq\,.$$

An unfortunate slip occurs on page 36. By simply taking the sense of the inequality $n-r\leqq r$ to be $n-r\geqq r$, certain incorrect conclusions are drawn. These errors are to be corrected at once by an erratum slip. The chapter on mean values and dispersion seems to the reviewer to be one of the most interesting chapters of the book, particularly because it presents the author's extension of the Lexis theory to mean values of observations