A TRIVIAL TAUBERIAN THEOREM.*

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The name Tauberian was introduced by Hardy† to describe a very interesting type of theorem in connection with summable series; he and others have enunciated a considerable number of such theorems bearing on various specific definitions of summability. We may indicate the general character of a Tauberian theorem as follows. The ordinary questions on summability consider two related sequences (or other functions) and ask whether it will be true that one sequence possesses a limit whenever the other possesses a limit, the limits being the same; a Tauberian theorem appears, on the other hand, only if this is untrue, and then asserts that the one sequence possesses a limit provided the other sequence both possesses a limit and satisfies some additional condition restricting its rate of increase. The interest of a Tauberian theorem lies particularly in the character of this additional condition, which takes different forms in different cases. Thus, if the condition is to be imposed on the term u_n of a series, it may take any of the forms (for which I write alternative notations)

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\begin{array}{ll}
|u_n| < Kf_n : & u_n = O(f_n)\ , \\
u_n < Kf_n : & u_n = O_+(f_n)\ , \\
u_n > -Kf_n : & u_n = O_-(f_n)\ , \\
u_n/f_n \to 0 : & u_n = o(f_n)\ , \\
\limsup_{n\to\infty} u_n/f_n = 0 : & u_n = o_+(f_n)\ , \\
\liminf_{n\to\infty} u_n/f_n = 0 : & u_n = o_-(f_n)\ ;
\end{array}
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† With respect to the name, see Hardy and Littlewood, PROCEEDINGS OF THE LONDON SOCIETY, vol. 2, (1912-13), p. 1.