COOLIDGE ON COMPLEX GEOMETRY

The Geometry of the Complex Domain. By Julian Lowell Coolidge. New York, Oxford University Press, American Branch, 1924. 242 pp.

The various geometries of a complex space are, with certain definite exceptions, essentially the same as the corresponding geometries of a real space, provided that one agrees, tacitly or wittingly, to admit to consideration only those configurations in complex space which are the direct generalizations of configurations in real space in that they arise from these by the replacement of real by complex numbers. This agreement restricts, for example, the discussion of manifolds of complex points to those manifolds depending on a fixed number of complex parameters. But a system of points depending on n complex parameters is but a very special case of a system depending on the equivalent number, $2n$, of real parameters, and its properties are in no way indicative of those of the general $2n$-parameter system. In other words, when the scope of complex geometry is widened to cover all possibilities, it takes on an entirely different aspect from that of real geometry.

Though there are recent books on complex geometry which pursue the subject along the narrow path outlined by the geometry of reals, the one now under review is the first to follow consistently the wider and, for the progress of mathematics, the more important, point of view. It aims to bring the reader abreast of the times in the advances, from this point of view, of the last half-century. The pioneers in these researches have been Segre and Study, and to them the book is informally dedicated. Among their disciples are to be mentioned Autonne, Benedetti, Coolidge, Fubini, Loewy, Sforza, and J. W. Young.

Closely connected with, if not strictly speaking a part of, complex geometry is the classical problem of representing complex points by real elements. An historical and critical account of the many attempts, ancient and modern, successful and unsuccessful, to solve this problem forms the second integral part of the book. The writers here are legion; we content ourselves with the names of the more important: Wallis, Wessel, Argand, Gauss, Poncelet, von Staudt, Laguerre, Marie, Klein, Segre, and Study.

The material has been well digested and excellently ordered. The complex line (Chapters I, II), the complex plane (Chapters III–VI), and complex three-dimensional space (Chapter VII) are taken up in turn. In each case, the real representations of complex points are first considered and then later put to work in throwing light on the complex geometry itself. The author writes in his characteristic, red-blooded style; his historical accounts and criticisms are, in particular, enter-