THE TENSOR CHARACTER OF THE GENERALIZED KRONECKER SYMBOL*

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1. *Introduction.* In a previous paper† we have considered the use of the generalized Kronecker symbol $\delta^{r_1 r_2 \cdots r_m}_{s_1 s_2 \cdots s_m}$ in presenting the theory of determinants and we now proceed to show that it is an arithmetic tensor of the type indicated by its subscripts and superscripts, i. e., it is covariant of rank m and contravariant of rank m. By the statement that a tensor is arithmetic, we mean that its presentation is independent of the particular coordinate system in use and that it has the same numerical values for its various components at all points of space.‡

The generalized Kronecker symbol may be defined by means of the equation

$$\delta^{r_1 r_2 \cdots r_m}_{s_1 s_2 \cdots s_m} = \begin{vmatrix} \delta^{r_1}_{s_1} & \cdots & \delta^{r_1}_{s_m} \\ \delta^{r_2}_{s_1} & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \delta^{r_m}_{s_1} & \cdots & \delta^{r_m}_{s_m} \end{vmatrix}. \tag{1}$$

Here the labels r and s run independently over a set of n numbers $1, 2, \cdots, n$ and $\delta^r_s = 1$ if $r = s$, and $= 0$ if $r \neq s$; δ^r_s is the ordinary Kronecker symbol and it is usually denoted by g^r_s in the theory of relativity. It is there derived as the scalar product of the metric tensor g_{rs} and its reciprocal g^{rs}, but this mode of presentation is somewhat

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† *The Generalized Kronecker symbol and its application to the theory of determinants,* AMERICAN MATHEMATICAL MONTHLY vol. 32 (1925), p. 233. This paper will be denoted by the symbol (A) in references below.

‡ Cf. P. FRANKLIN, PHILOSOPHICAL MAGAZINE, (6), vol. 45 (1923), p. 998.