THE NUMBER OF EVEN AND ODD ABSOLUTE PERMUTATIONS OF n LETTERS*

BY J. M. THOMAS†

A permutation of n letters is called absolute if it leaves no letter fixed. A formula for the total number of such permutations is well known, but no separate formulas for the numbers of even and odd absolute permutations seem to be given in the literature. It is the purpose of this note to show how such formulas can be derived.

Consider the determinant of order n

$$D = \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & 1 & \cdots & 0 \end{vmatrix},$$

all of whose elements are equal to 1, except those on the main diagonal, which equal 0. By writing it in the form

$$\begin{vmatrix} 1-1, & 1+0, & 1+0, & \cdots, & 1+0 \\ 1+0, & 1-1, & 1+0, & \cdots, & 1+0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1+0, & 1+0, & 1+0, & \cdots, & 1-1 \end{vmatrix},$$

we see that it is expressible as the sum of 2^n determinants which fall into three classes. Those determinants having more than one column of 1's are zero. Those having exactly one column of 1's are n in number, and their common value is $(-1)^{n-1}$. Finally there is just one determinant containing no column of 1's, and its value is $(-1)^n$. Hence the value of D is $(-1)^{n-1}(n-1)$.

On the other hand if we consider the expanded form of D, say

$$D = \sum \pm a_{1i} a_{2j} \cdots a_{nk},$$

---

* Presented to the Society, February 28, 1925.

† National Research Fellow in Mathematics.