REDUCTION OF EULER'S EQUATIONS TO A CANONICAL FORM*

BY J. H. TAYLOR†

1. *Introduction.* As a by-product of the preparation of an earlier paper by the author, *A generalization of Levi-Civita's parallelism and the Frenet formulas*, Dissertation, University of Chicago, 1924,‡ two useful methods of solving for the second derivatives in Euler's equations associated with the problem of minimizing an integral were discovered. In this paper these two methods are presented in detail. Whereas it appears at first that the assumptions required to effect the solution in the two instances are quite different, it is here shown that in each of the two cases the assumptions which are made may be replaced by the supposition that the F_1 function of the calculus of variations does not vanish.

2. *The Euler Equations.* Suppose that

$$x^\alpha = x^\alpha(u), \quad u_1 \leq u \leq u_2, \qquad (\alpha = 1, \cdots, n),$$

are the equations of one of a class of curves joining two fixed points in an n-space, and let us consider the problem of selecting that curve of the class which gives to the integral

$$I = \int_{u_1}^{u_2} F(x, x')du$$

its minimum value. Here x and $x' = dx/du$ stand for the sets $x^1, \cdots, x^n$ and $x'^1, \cdots, x'^n$ respectively. It will be assumed that F satisfies the homogeneity condition

$$F(x, \varkappa x') = \varkappa F(x, x'), \qquad \varkappa > 0. \tag{1}$$

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† National Research Fellow in Mathematics.

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