SIMPLIFICATIONS RELATING TO A PROOF OF SYLOW'S THEOREM

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Sylow's theorem is usually developed very early in a course relating to the theory of groups of finite order. Hence simplifications in its proof are the more desirable, especially when they enable us to avoid the use of a number theory formula and therefore to confine the proof more closely to group theoretic considerations. Since the simplifications which will be developed in what follows involve a property of double cosets which is not directly used in the proof to which they relate* we shall first exhibit some of the fundamental properties of double co-sets, assuming as known the fact that if H_1 and H_2 are any two subgroups of a group G then all the operators of G may be uniquely represented in the following form. $G = H_1s_1H_2 + \cdots + H_1s_\lambda H_2$ if we assume that in each double co-set only the distinct operators are considered.

The number of distinct operators in each double co-set is evidently divisible by h_1 and by h_2, h_1 and h_2 being the orders of H_1 and H_2 respectively. A necessary and sufficient condition that each of the given double co-sets contains the same number of distinct operators is that each of the conjugates of H_1 under G has the same number of operators in common with H_2. This is equivalent to saying that each of these conjugates of H_2 has the same number of operators in common with H_1. When this number of common operators is k then the number of distinct operators in each of these co-sets is h_1h_2/k, and vice versa. In particular, each of these double co-sets must involve the same number of distinct operators whenever at least one of the two subgroups H_1, H_2 is invariant under G, and each of the double co-sets with respect

* Miller, Blichfeldt, and Dickson, *Finite Groups*, 1916, p. 27.