NOTE ON A CLASS OF HARMONIC FUNCTIONS

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1. *Introduction.* Let S be a multiply-connected open region, bounded by an exterior circle s_0 and n interior circles $s_1, \cdots, s_n$, not touching, and let $u(P)$ be harmonic, not necessarily bounded, in S. The author has shown that a necessary and sufficient condition that $u(P)$ be given as the sum of logarithmic terms and Poisson-Stieltjes integrals around the several boundaries is that the integrals $\int_0^{2\pi} |u| d\theta_i$, extended around circles concentric with each boundary circle, remain bounded in the neighborhood of each boundary circle. He has shown also that a necessary and sufficient condition that $u(P)$ be given by the corresponding Stieltjes integral generalization of the formula in terms of the normal derivative of the Green's function is that $\int |u| dh$ remain bounded, where the integration is extended over the whole of a variable curve $g = \text{const.}$, with g the Green's function and h its conjugate, referred to some fixed pole Q. Finally the author has shown that a necessary and sufficient condition for the latter formula is that $u(P)$ be the difference of two functions harmonic and not-negative in S.* It is desired to show that this condition is also necessary and sufficient for the former mode of representation, so that the two forms of representation apply to the same class of functions. That the condition is necessary follows at once from the mode of representation. It is only the sufficiency that requires proof.

2. *Lemma* I. Let S_0 be a simply-connected open region, bounded internally or externally by a circle s_0, of radius

* *The Dirichlet problem for the general open finitely-connected region,* PROCEEDINGS OF THE INTERNATIONAL CONGRESS AT TORONTO (1924).